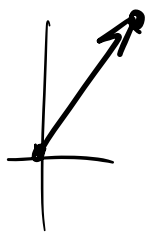


Due Sun

3.1 – Vectors in 2-Space, 3-Space, and n -Space

Definition 1: If n is a positive integer, then an **ordered n -tuple** is a sequence of n real numbers (v_1, v_2, \dots, v_n) . The set of all ordered n -tuples is called **real n -space**, denoted by R^n .

(2, 3)

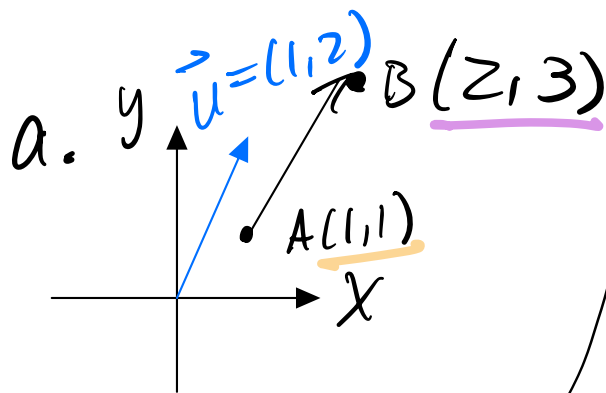


If a vector \mathbf{v} in 2-space or 3-space is positioned with its initial point at the origin of a rectangular coordinate system, then the coordinates of its terminal point are **components** of \mathbf{v} relative to the coordinate system. The numbers in an n -tuple can be considered coordinates of a point or components of a vector.

5. a. Find the terminal point of the vector that is equivalent to

$\mathbf{u} = (1, 2)$ and whose initial point is $A(1, 1)$.

b. Find the initial point of the vector that is equivalent to $\mathbf{u} = (1, 1, 3)$ and whose terminal point is $B(-1, -1, 2)$.



$$\vec{u} = (2-1, 3-1)$$

terminal - initial

b. want $A(x_1, y_1, z_1)$
such that $\vec{AB} = \vec{u}$

so

$$(-1-x_1, -1-x_2, 2-x_3) = (1, 1, 3)$$

$$-1-x_1 = 1 \Rightarrow x_1 = -2$$

$$-1-x_2 = 1 \Rightarrow x_2 = -2$$

$$2-x_3 = 3 \Rightarrow x_3 = -1$$

$$\underline{A(-2, -2, -1)}$$

7. Find the initial point P of a nonzero vector $\mathbf{u} = \overrightarrow{PQ}$ with terminal point $Q(3, 0, -5)$ and such that

a. \mathbf{u} has the same direction as $\mathbf{v} = (4, -2, -1)$.

b. \mathbf{u} is oppositely directed to $\mathbf{v} = (4, -2, -1)$.

a.

$\vec{v} = (4, -2, -1)$

$Q(3, 0, -5)$

$P(x_1, y_1, z_1)$ \vec{u}

$4 = 3 - x_1$
 $-2 = -y_1$
 $-1 = -5 - z_1$

$P(-1, 2, -4)$

b.

$\vec{v} = (4, -2, -1)$

$-\vec{v} = (-4, 2, 1)$

$Q(3, 0, -5)$

$P(x_2, y_2, z_2)$

$-4 = 3 - x_2 \Rightarrow x_2 = 7$

$+2 = -y_2 \Rightarrow y_2 = -2$

$+1 = -5 - z_2 \Rightarrow z_2 = -6$

$P(7, -2, -6)$

Definition 3: If $\mathbf{v} = (v_1, v_2, \dots, v_n)$ and $\mathbf{w} = (w_1, w_2, \dots, w_n)$ are vectors in R^n , and if k is any scalar, then we define

- $\mathbf{v} + \mathbf{w} = (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n)$
[this is **vector addition**]
- $k\mathbf{v} = (kv_1, kv_2, \dots, kv_n)$
[this is **scalar multiplication**]
- $-\mathbf{v} = (-v_1, -v_2, \dots, -v_n)$
- $\mathbf{w} - \mathbf{v} = \mathbf{w} + (-\mathbf{v}) = (w_1 - v_1, w_2 - v_2, \dots, w_n - v_n)$

12. Let $\mathbf{u} = (1, 2, -3, 5, 0)$, $\mathbf{v} = (0, 4, -1, 1, 2)$, and $\mathbf{w} = (7, 1, -4, -2, 3)$. Find the components of

a. $\mathbf{v} + \mathbf{w}$

b. $3(2\mathbf{u} - \mathbf{v})$

c. $(3\mathbf{u} - \mathbf{v}) - (2\mathbf{u} + 4\mathbf{w})$

d. $\frac{1}{2}(\mathbf{w} - 5\mathbf{v} + 2\mathbf{u}) + \mathbf{v}$

$$3\vec{u} - \vec{v} =$$

$$c. (3, 6, -9, 15, 0) - (0, 4, -1, 1, 2)$$

$$= (3, 2, -8, 14, -2)$$

The **zero vector**, $\mathbf{0} = (0, 0, \dots, 0)$, has length zero.

$$\mathbb{R}^2: \vec{0} = (0, 0), \quad \mathbb{R}^3: \vec{0} = (0, 0, 0)$$

Theorem 3.1.1 If \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors in \mathbb{R}^n , and if k and m are scalars, then:

a) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

b) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}) = \mathbf{u} + \mathbf{v} + \mathbf{w}$

c) $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$

d) $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

e) $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$

f) $(k + m)\mathbf{u} = k\mathbf{u} + m\mathbf{u}$

g) $k(m\mathbf{u}) = (km)\mathbf{u}$

h) $1\mathbf{u} = \mathbf{u}$

Pf (A): Let $\vec{u} = (u_1, u_2, \dots, u_n) \in \mathbb{R}^n$ and $k, m \in \mathbb{R}$

$$(k+m)\vec{u} = (k+m)(u_1, u_2, \dots, u_n)$$

$$= ((k+m)u_1, (k+m)u_2, \dots, (k+m)u_n)$$

(by def of scalar mult.)

$$= (ku_1 + mu_1, ku_2 + mu_2, \dots, ku_n + mu_n)$$

(by dist. prop. of mult of real #s)

$$= (ku_1, ku_2, \dots, ku_n) + (mu_1, mu_2, \dots, mu_n)$$

(def of vector addition)

$$= k(u_1, u_2, \dots, u_n) + m(u_1, u_2, \dots, u_n)$$

(def of scalar multiplication)

$$= k\vec{u} + m\vec{u} \quad \checkmark$$

Theorem 3.1.2 If \mathbf{v} is a vector in \mathbb{R}^n and k is a scalar, then:

a) $0\mathbf{v} = \mathbf{0}$

b) $k\mathbf{0} = \mathbf{0}$

c) $(-1)\mathbf{v} = -\mathbf{v}$

Definition 4: if \mathbf{w} is a vector in R^n , then \mathbf{w} is said to be a **linear combination** of the vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r$ in R^n if it can be expressed in the form $\mathbf{w} = k_1\mathbf{v}_1 + k_2\mathbf{v}_2 + \dots + k_r\mathbf{v}_r$, where each k_i is a scalar. The scalars are called the **coefficients** of the linear combination. In the case where $r = 1$, we have $\mathbf{w} = k_1\mathbf{v}_1$, in which case the linear combination is a scalar multiple of the vector.

Check: $3\vec{u} - \vec{v} = (3, -3, 9, 15) + (-2, -1, 0, 3) = \underline{(1, -4, 9, 18)}$

17. Let $\mathbf{u} = (1, -1, 3, 5)$ and $\mathbf{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\mathbf{u} + b\mathbf{v} = \underline{(1, -4, 9, 18)}$.

$$a(1, -1, 3, 5) + b(2, 1, 0, -3) = (1, -4, 9, 18)$$

$$\begin{bmatrix} 1 \\ -1 \\ 3 \\ 5 \end{bmatrix} a + \begin{bmatrix} 2 \\ 1 \\ 0 \\ -3 \end{bmatrix} b = \begin{bmatrix} 1 \\ -4 \\ 9 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \\ 3 & 0 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 9 \\ 18 \end{bmatrix} \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ -1 & 1 & -4 \\ 3 & 0 & 9 \\ 5 & -3 & 18 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right] \rightarrow \begin{matrix} a = 3 \\ b = -1 \end{matrix}$$

A contradiction here means there are no such a & b .